Spontaneous Symmetry Breaking in Gauge Theories

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Outline of the Presentation

Introduction and Motivation

Symmetry Breaking—A Natural Phenomenon?

Local Gauge Symmetry Breaking?

SSB of Global Subgroups

Philosophical Implications
Interpretation of (gauge) symmetries and (gauge) symmetry breaking: central topic in the philosophy of science.

Gauge symmetries: “purely formal” (Healey), no physical instantiations.

Received view: SSB of local gauge symmetry crucial for the Higgs mechanism.
Criticisms of the received view

- ’t Hooft: SSB of local gauge symmetry “something of a misnomer”
- Earman: should use constraint Hamiltonian formalism to clarify whether there is any SSB in the Higgs mechanism
- Lyre: There is no “Higgs mechanism”.
- Elitzur (1975): Local gauge symmetry cannot be spontaneously broken (rigorous result).

Let’s take a closer look!
SSB and symmetry breaking order parameters

- Basic idea of SSB: State not transformable into one with the same symmetries as the underlying “laws”.
- Need infinitely many degrees of freedom.
- In quantum theories: Unitary inequivalence of representation associated to different symmetry breaking states.
- ⇒ Need “more than one” Hilbert space.
- If \( \langle A \rangle \neq \langle \alpha(A) \rangle \), then \( \langle A \rangle \) is called a “symmetry breaking order parameter” for the symmetry \( \alpha \).
SSB as a natural phenomenon

- SSB is something that “occurs”.
- It applies to (the states of) objects themselves, not merely our description.
- Liu and Emch (2005) conclude: SSB is a “natural phenomenon”...
- ... in contrast to mere “theoretical concepts” such as “renormalization” or “quantization”.

But what about gauge SSB?
Gauge SSB

- Gauge SSB not a “natural phenomenon” in the sense of *asymmetric reality governed by symmetric laws*.
- But: Gauge SSB possibly a “natural phenomenon” in a (slightly) weaker sense:
- Transition from unbroken to broken gauge symmetry may indicate a qualitative change in observable properties.
- In *that* sense gauge SSB can be called a “natural phenomenon” if it signals a phase transition.
- Spontaneous breaking of an observable symmetry *always* corresponds to a phase transition.
BEC as an example

Example of gauge SSB: Bose-Einstein condensation (BEC)

- Broken global gauge symmetry.
- Distinction between broken and unbroken symmetry lines up with phase transition.
- Macroscopic observables different in the two phases (e.g. compressibility, viscosity, specific heat).
Local gauge symmetry: classical case

Consider

$$\mathcal{L} = D_\mu \phi^* D^\mu \phi - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$  \hspace{1cm} (1)

with (Abelian) local $U(1)$ gauge symmetry:

$$\phi(x) \mapsto e^{i\alpha(x)} \phi(x), \hspace{1cm} A_\mu(x) \mapsto A_\mu(x) - \frac{1}{e} \partial_\mu \alpha(x)$$  \hspace{1cm} (2)

and potential:

$$V(\phi) = m_0^2 \phi^* \phi + \lambda_0 (\phi^* \phi)^2,$$  \hspace{1cm} (3)

where $m_0^2 < 0$.

Classical ground states:

$$\phi(x) = e^{i\theta(x)} v / \sqrt{2}$$  \hspace{1cm} (4)

$\Rightarrow$ SSB of local gauge symmetry.
Generating functional:

\[ W[\eta, J] = N \int D\phi D A_\mu \exp \left( i \int d^4x (\mathcal{L} + \eta \phi + J_\mu A^\mu) \right). \quad (5) \]

- The integral in Eq. (5) is “badly divergent”.
- No perturbation expansion possible.
- Either: Quantize non-perturbatively on a lattice.
- Or: Fix the gauge.
Without gauge fixing:

*Local gauge symmetry cannot be spontaneously broken.* (Elitzur, 1975)

Crux of proof: Can’t implement nonzero order parameter by fixing it on a finite volume boundary.

Reason: Gauge transformations depend on infinitely many parameters.
Roughly: Replace the action $S$ in the functional integral $W[\eta, J]$ by an “effective action”

$$S_{\text{eff}} = S + S_{gf} + S_{\text{ghost}},$$

where $S_{gf}$ breaks local gauge invariance.
Post-gauge fixing global symmetries

$S_{gf}$ may either break local gauge invariance completely or not.

- If so: Only one configuration per gauge orbit compatible with gauge fixing condition.
- If not: $S_{gf}$ still invariant under remnant global gauge symmetry.

Examples:
- Unitary gauge: $\theta(x) = 0$
- Coulomb gauge, Landau gauge, temporal gauge,...

Remnant global gauge symmetries may break spontaneously!
Global gauge SSB and phase transitions I

Question: Are these global gauge SSBs always associated with phase transitions?

Answer: No!
Whether remnant global gauge symmetry is broken or unbroken depends on the gauge.

Example: Different unbroken/broken transitions in Landau gauge and Coulomb gauge for an $SU(2)$-Higgs model (Caudy and Greensite, 2008)

No rigid connection between phase transitions and global gauge SSBs.

$\Rightarrow$ Global gauge SSB is “ambiguous”, not (in general) a “natural phenomenon’.
Kosso: Gauge symmetry breakings belong to the class of cases where “the relevant laws of nature are exactly symmetric, but the phenomena expressing these laws are not.” (Kosso 2000)

Criticism:

- broken gauge symmetry: No asymmetry in observable phenomena (see Bose-Einstein condensation).
- in gauge theories: breaking of global subgroups is *ambiguous*. 
Morrison on “vacuum hypotheses”

Morrison: “it would be folly to accept a robust physical interpretation of the SSB story.” (Morrison 2003)
Reasonable conclusion, but based on confused reasoning (“theoretical story about the nature of the vacuum”):

Criticism:

- Without gauge fixing: vacuum expectation value is zero (Elitzur).
- With gauge fixing: vacuum expectation value depends on the gauge and...
- ...especially, whether or not it is zero depends on the gauge.
Weinberg on “reality of gauge symmetries”

Weinberg: “if a gauge symmetry becomes unbroken for sufficiently high temperature, it becomes difficult to doubt its reality.” (Weinberg 1978)

Whatever “reality” is supposed to mean:

- No evidence against standard view of gauge symmetries as purely formal!
- Gauge symmetry “restoration” depends on the gauge, is ambiguous.

⇒ No indication for “reality” of gauge symmetries in whatever sense.