Gauge Symmetry Breaking—An Attempt to Clarify

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Motivation: What is gauge symmetry breaking?

- Received view: Spontaneous breaking of local gauge symmetry crucial for Higgs mechanism (but not uncontested).
- Interpretation of (gauge) symmetries and (gauge) symmetry breaking: central topic in the philosophy of science.
- Gauge symmetries: "purely formal" (Healey), no physical instantiations.
- "If gauge symmetry merely indicates descriptive redundancy in the mathematical formalism, it is not clear how spontaneously breaking a gauge symmetry could have any physical consequences, desirable or not." (Smeenk 2006)

Aim: Clarify status of SSB in gauge theories

Outline of the Presentation

1. Introduction:
2. Local Gauge Symmetry Breaking?
3. Remnant Global Symmetries
4. Philosophical Implications

SSB and symmetry breaking order parameters

- Basic idea of SSB: States need not have all the symmetries of the underlying "laws".
- Need infinitely many degrees of freedom.
- In quantum theories: Unitary inequivalence of representations associated with different symmetry breaking states.
- $\Rightarrow$ Need "more than one" Hilbert space.
- If $\langle A \rangle \neq \langle \alpha(A) \rangle$, then $\langle A \rangle$ is called a "symmetry breaking order parameter" for the symmetry α.

SSB and phases

- Liu and Emch (2005): SSB is a “natural phenomenon”...
- ... in contrast to mere “theoretical concepts” such as “renormalization” or “quantization”.
- In which sense? Contrast between broken and unbroken symmetry corresponds to distinction between phases!
- Zero or nonzero value of $\langle A \rangle$ distinguishes the phases.

Gauge SSB

But what about gauge SSB in gauge theories?

- Does it exist? If so, in what sense?
- A gauge-dependent order parameter like $\langle \phi \rangle$ (Higgs field) is unobservable.
- So: Does the contrast between broken and unbroken gauge symmetry correspond to a contrast between phases?

BEC as an example

Example of gauge SSB: Bose-Einstein condensation (BEC)

- Broken **global** gauge symmetry.
- Distinction between broken and unbroken symmetry lines up with phase transition.
- Macroscopic observables different in the two phases (e. g. compressibility, specific heat).

Local gauge symmetry: classical case

Consider, as an example,

$$\mathcal{L} = D_\mu\phi^* D^\mu\phi - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \tag{1}$$

with (Abelian) local $U(1)$ gauge symmetry:

$$\phi(x) \mapsto e^{i\alpha(x)}\phi(x), \qquad A_\mu(x) \mapsto A_\mu(x) - \frac{1}{e}\partial_\mu\alpha(x) \tag{2}$$

and potential:

$$V(\phi) = m_0^2\phi^*\phi + \lambda_0(\phi^*\phi)^2\,, \tag{3}$$

where $m_0^2 < 0$.

Classical ground states:

$$\phi(x) = e^{i\theta(x)}v/\sqrt{2} \tag{4}$$

$\Rightarrow$ SSB of local gauge symmetry.

Unitary gauge

As an aside:

Writing $\phi(x) = e^{i\theta(x)}\rho(x)$ and implementing the unitary gauge

$$\theta(x) = 0 \tag{5}$$

(textbook expositions) eliminates the gauge freedom completely.

$\Rightarrow$ No SSB possible any more.

Arguably: Textbook exposition ("reshuffling of degrees of freedom") only useful to extract physical degrees of freedom, otherwise misleading.

Local gauge symmetry: Quantization

Generating functional:

$$W[\eta, J] = N \int \mathcal{D}\phi \mathcal{D}A_\mu \exp\left(i \int \mathrm{d}^4\mathrm{x}(\mathcal{L} + \eta\phi + \mathrm{J}_\mu \mathrm{A}^\mu) \right) . \tag{6}$$

- The integral in Eq. (6) is “badly divergent”.
- No perturbation expansion possible.
- Either: Quantize non-perturbatively on a lattice (Wilsonian formulation).
- Or: Fix the gauge.

Without gauge fixing

Elitzur's theorem:

Local gauge symmetry cannot be spontaneously broken. (Elitzur, 1975)

Crux of proof: Can't implement nonzero order parameter by fixing it on a finite volume boundary.

Reason: Gauge transformations depend on infinitely many parameters.

Generalization: Only gauge-invariant variables can have nonzero expectation values.

What about the electroweak phase transition?

According to common wisdom:

- Electroweak phase transition occurs in early universe,
- corresponds to spontaneous breakdown of local gauge symmetry,
- expectation value of the Higgs functions as order parameter.

Elitzur:

Must characterize phase transition differently, for instance through $\langle \phi^* \phi \rangle$:
"non-symmetry breaking order parameter", jumps at phase transition.
See, for instance, (Buchmüller et al., 1994), "Gauge invariant treatment of the electroweak phase transition"

Gauge fixing

Roughly: Replace the action S in the functional integral $W[\eta, J]$ by an "effective action"

$$S_{eff} = S + S_{gf} + S_{ghost} \,, \tag{7}$$

where S_{gf} breaks local gauge invariance.

Post-gauge fixing global symmetries

S_{gf} may either break local gauge invariance *completely* or not.

- If so: Only one configuration per gauge orbit compatible with gauge fixing condition.
- If not: S_{gf} still invariant under remnant **global** gauge symmetry.

Examples:

- Unitary gauge: $\theta(x) = 0$
- Coulomb gauge, Landau gauge, temporal gauge,...

Remnant global gauge symmetries may break spontaneously.

But what does their breaking mean???

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Global gauge SSB and phase transitions I

Question: Are these global gauge SSBs alwas associated with phase transitions?

Answer: No!

Global gauge SSB and phase transitions II

Whether remnant global gauge symmetry is broken or unbroken depends on the gauge.

Example: Different unbroken/broken transitions in Landau gauge and Coulomb gauge for an $SU(2)$-Higgs model (Caudy and Greensite, 2008). Both don't generally coincide with phase transition (Fradkin and Shenker, 1978).

No rigid connection between phase transitions and global gauge SSBs.

$\Rightarrow$ Global gauge SSB is "ambiguous", **not** (in general) a "natural phenomenon'.

Kosso on epistemology of SSB

Kosso: Gauge symmetry breakings belong to the class of cases where “the relevant laws of nature are exactly symmetric, but the phenomena expressing these laws are not.” (Kosso 2000)

Criticism:

- broken gauge symmetry: No asymmetry in observable phenomena (see Bose-Einstein condensation).
- in gauge theories: breaking of global subgroups is *ambiguous*.

Morrison on “vacuum hypotheses”

Morrison: “it would be folly to accept a robust physical interpretation of the SSB story.” (Morrison 2003)
Reasonable conclusion, but based on confused reasoning (“theoretical story about the nature of the vacuum”):

Criticism:

- Without gauge fixing: vacuum expectation value is zero (Elitzur).
- With gauge fixing: vacuum expectation value depends on the gauge and...
- ...especially, whether or not it is zero depends on the gauge.

Weinberg on “reality of gauge symmetries”

Weinberg: “if a gauge symmetry becomes unbroken for sufficiently high temperature, it becomes difficult to doubt its reality.” (Weinberg 1978)

Whatever “reality” is supposed to mean:

- No evidence against standard view of gauge symmetries as purely formal!
- Gauge symmetry “restoration” depends on the gauge, is ambiguous.

$\Rightarrow$ In particular: no indication for “reality” of gauge symmetries in the sense of observability.